

Wage Flexibility and the Optimal Inflation Rate

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Abstract

In this paper we develop a model of wage indexation from basic principles. One of the implications of this model is that if wages are downwardly rigid, partial indexation of wages is optimal. Under such conditions the socially optimal inflation rate is greater than zero.

1 Introduction

Throughout the OECD countries, wages are set through negotiations between management and union representatives. Under such agreements wages are fixed at a nominal value or are increased at an agreed amount per year, for a given contract period. At the termination of this period it is agreed that negotiations will be made for new wage contracts.

In Tobin's A.E.A. presidential address (1972) he postulated an economy where wage contracts had a floor on wage changes as nominal wage cuts were resisted by labor. Tobin attributed this resistance to workers' concern for wages relative to workers in other industries. This emphasis on relative wage increases meant that inflation could serve a socially useful purpose by keeping real wages down. Tobin opines¹:

No one has devised a way of controlling average wage rates without intervening in the competitive struggle over relative wages. Inflation lets this struggle proceed and blindly, impartially, impersonally, and nonpolitically scales down all

¹Tobin (1972), p. 13.

its outcomes. There are worse methods of resolving group rivalries and social conflict.

Recently this idea has been taken up in the work of Akerlof, Dickens and Perry (1996) and Card and Hyslop (1996).

In this paper we study the form of optimal wage contracts with endogenously determined contract length and indexation. A form of wage contract will be assumed in which contracts specify the initial wage and the rate of wage increase over time. The model in Section 2 gives a very simple closed form solution for optimal contract length under conditions of complete wage flexibility. In this model, wages are fully indexed to the expected rate of inflation. The result derived in this section is identical to that of Grey (1978).

In Section 3 we take up Tobin's conjecture and show that if wages are slightly downwards inflexible, then partial indexation with respect to expected inflation is optimal. If wages are not allowed to decrease over the term of a contract, then an inflation rate greater than zero is socially optimal.

2 Bargaining model with flexible wages

We assume the firm and union have inverse labor demand and supply functions (respectively) of the form:

$$\text{Supply: } w_t = a + cL_t + \epsilon_t \quad (1)$$

$$\text{Demand: } w_t = b - dL_t + \nu_t \quad (2)$$

where w_t is the nominal wage rate and L_t labor. We assume that ν_t and ϵ_t are Brownian motions with drift with the distribution:

$$\begin{bmatrix} \epsilon_t \\ \nu_t \end{bmatrix} \sim N \left(\begin{bmatrix} et \\ ft \end{bmatrix}, \begin{bmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\nu}^2 \\ \sigma_{\epsilon\nu}^2 & \sigma_\nu^2 \end{bmatrix} t \right)$$

These shocks will typically include the (predicted and unpredicted) effects of inflation, shocks to labor productivity and demand shocks. It is expected that some components of the variance of ν and ϵ , such as inflation, will be common to both shocks. This will enter in the covariance term. Given that these shocks are Brownian motions, equilibrium wages follow a Brownian motion with drift of the form

$$w_t \sim N(w_0 + \mu t, \sigma^2 t)$$

where we define

$$w_0 \equiv \frac{ad+bc}{c+d} \quad (3)$$

$$\mu \equiv \frac{cf+de}{c+d} \quad (4)$$

$$\sigma^2 \equiv \frac{c^2\sigma_\nu^2+2cd\sigma_\nu\sigma_\epsilon+d^2\sigma_\epsilon^2}{c+d} \quad (5)$$

Inflation will enter into this model in two ways. Expected inflation will raise equally ν and ϵ and enter through the parameters e and f . Unexpected monetary shocks will enter through changes in variance and covariance terms.

We assume that nominal wages are set in advance over the length of a contract between representatives of labor (the “union”) and representatives of the firm (the “firm”). These contracts will specify the contract wage as a function of time. Over the duration of the contract, shocks to the labor supply and demand schedules mean that the wage set under the contract will not necessarily be equal to the equilibrium wage.

In this case we assume that the market will be cleared by the short side of the schedules. That is, when the contract wage falls below the equilibrium wage, labor is determined by the labor supply schedule, and when the contract wage is above the equilibrium wage, labor is determined by the labor demand schedule. Wages are fixed in the short-run, but labor is supplied and consumed on the spot market. This is accomplished in economies with mandated work-weeks by such measures as firms varying the level of overtime, the rate of filling job vacancies and temporary factory shut-downs.

Given that the wage negotiation is a repeated game between the firm and the union, we assume that the two parties will negotiate so as to minimize their joint loss. Disparities in bargaining power can then be settled with side-payments between the parties. The joint loss of the firm and the union under the contract wage is the usual welfare triangle, assuming away any wealth effects. If \bar{w}_t is the contracted wage rate and w_t is the equilibrium wage rate at date t , then total welfare loss can be measured by

$$\frac{1}{2}\Delta_s(w_t - \bar{w}_t)^2 \quad \text{if } w_t > \bar{w}_t \quad (6)$$

$$\frac{1}{2}\Delta_d(w_t - \bar{w}_t)^2 \quad \text{if } w_t < \bar{w}_t \quad (7)$$

where Δ_s is the slope of the supply curve and Δ_d is the slope of the

demand curve².

Negotiations between the union and management have a cost, C , which will include research on the current and future state of the market; negotiation posturing, such as labor strikes or factory shut-downs; and costs of holding the meetings of the representatives of the two sides. Over a given length of time, T , if the contracts have a duration, T^* , there will have to be $n = \frac{T}{T^*}$ negotiations at a combined cost of nC ³.

The expected joint loss over the duration of the contract due to the deviation of equilibrium wage from the contract wage is

$$E_0 \left\{ \int_0^{T^*} \frac{1}{2} (\text{Prob}(w_t > \bar{w}_t)\Delta_s + \text{Prob}(w_t < \bar{w}_t)\Delta_d)(w_t - \bar{w}_t)^2 dt \right\} \quad (8)$$

Given that there are no asymmetries in the problem \bar{w}_t will be set equal to expected w_t in the optimal contract, so that

$$\bar{w}_t = w_0 + \mu t \quad (9)$$

and thus $E_0 \{\text{Prob}(w_t > \bar{w}_t)\} = \frac{1}{2} = E_0 \{\text{Prob}(w_t < \bar{w}_t)\}$. We set

$$\Delta = \frac{1}{2}(\Delta_s + \Delta_d) \quad (10)$$

As we assume that μ and σ are expected to be constant over the period $[0, T]$, the optimal duration of the contracts, T^* is constant over time.

Assuming that the management and the union minimize expected average loss over the time period of length T , the problem becomes one of choosing the optimal duration of the contracts, but this is equivalent to choosing the optimal number of renegotiations, n . The full minimization problem becomes:

$$\min_n \left\{ n \int_0^{\frac{T}{n}} \frac{1}{2} \Delta E_0 [(w_t - \bar{w}_t)^2] dt + nC \right\} \quad (11)$$

As $E_0 [(w_t - \bar{w}_t)^2]$ is the variance of a Brownian motion, it is equal to $\sigma^2 t$. Our minimization problem is reduced to

$$\min_n \left\{ \frac{n}{4} \Delta \sigma^2 \left(\frac{T}{n}\right)^2 + nC \right\} \quad (12)$$

²Note that we could alternatively assume that the slopes of the respective curves are linear approximations to non-linear curves in the region of the equilibrium.

³We will assume that T is large enough relative to T^* so that any remainder of $\frac{T}{T^*}$ can be ignored.

from which our first-order conditions give contract length of

$$\frac{T}{n} \equiv \sqrt{\frac{4C}{\Delta\sigma^2}} \quad (13)$$

From this we get the total loss over the period T of $2nC$ and given an average level of under- or unemployment over the contract of⁴

$$\frac{2}{3}\Delta\sigma\frac{T^{\frac{1}{2}}}{n} \quad (14)$$

so that the average level of unemployment over the contract is equal to

$$\frac{3}{\sqrt{2}}\sigma^{\frac{1}{2}}C^{\frac{1}{4}}\Delta^{\frac{3}{4}} \quad (15)$$

The results of this simple model accord with intuition. Contract length is increasing in renegotiation costs and is decreasing in the variance of equilibrium wages and the employment response of the wage variation (Δ). Average unemployment is increasing in all three of the above variables. Unemployment is neutral in the rate of inflation, as wages are fully indexed to expected inflation, but responds to unexpected variations in inflation, which will be a component of σ ⁵.

3 Bargaining model with inflexible wages

In this section we relax the assumption that wages contracts can specify any path for nominal wages. Instead we assume that workers are resistant to nominal decreases in wages. This will be modelled by increasing the negotiation costs of a negotiation that ends up reducing wages.

In the quote from Tobin in the introduction of the paper, Tobin conjectured that a positive level of inflation was optimal in the face of downward wage rigidities. We prove Tobin's conjecture in this model by showing that the optimal wage contracts will not be fully indexed to expected inflation. If the level of inflation is low enough the optimal contract would then require a declining nominal wage over the term of the contract. We assume that workers would resist a declining nominal

⁴Calculated given under- or unemployment is equal to $\Delta | w_t - \bar{w}_t |$

⁵But note that it has typically been found that the variance of inflation is positively correlated with the average level of inflation.

wage contract, so it is optimal to have an inflation rate high enough that the nominal wage is constant over the term of the contract.

As in the model in the above section we assume that contract negotiations are costly. But we modify the previous assumptions by allowing the cost of contract negotiations, in which contract wages are lowered from the last wage specified under the previous contract to the new contract, to differ from those contracts in which wages are raised. We assume that the cost of negotiations is C , if wages are raised or remain the same, but that the cost of a negotiation which lowers wages is γC , where $\gamma \geq 1$,

For the sake of simplicity, we assume that the slopes of the supply and demand curves are approximately equal, so that

$$\Delta \equiv \Delta_d \equiv \Delta_s \quad (16)$$

As the model has asymmetries, it is no longer the case that the optimal contract is continuous in time. This problem could be made identical to the problem in the previous section by setting

$$\bar{w}_t = w_0 + \mu t \text{ for } t \in [0..T) \text{ and } \bar{w}_T = 0 \quad (17)$$

In this case, as the wage is always increased between contracts, the cost of renegotiation is always C . A non-linear contract could be made to replicate this contract as closely as one wished. We do not see such complicated behavior in wage contracts, and so we will assume that the contracted wage be continuous and linear.

If we assume that the optimal contract is linear in time, at rate m , and that expected value of wages over the contract is equal to expected value of the equilibrium wages. This income constraint is

$$\int_0^{\frac{T}{n}} [\bar{w}_0 + mt] dt = E_0 \left\{ \int_0^{\frac{T}{n}} [w_0 + \mu t + \sigma dZ] dt \right\}$$

where σdZ is the standard deviation of the Brownian motion governing wages. Given that $E_0 dZ = 0$, this reduces to

$$(\bar{w}_0 - w_0) = \frac{1}{2}(\mu - m) \frac{T}{n}$$

Our minimization problem becomes

$$\min_{n,m} E_0 \left\{ n \int_0^{\frac{T}{n}} \left[\frac{1}{2} \Delta [w_0 + \mu t + \sigma dZ - \bar{w}_0 - mt]^2 \right] dt + nF(m)C + n(1 - F(m))\gamma C \right\}$$

subject to equation (3). Here $F(m)$ is the probability that the wage for date $\frac{T}{n}$ specified on the date 0 contract will be lower than the wage specified on the new contract at date $\frac{T}{n}$. $F(m)$ is the probability that the contracted wages will not fall at the end of the date 0 contract⁶. Temporarily denoting the specified wage under the period i contract by the superscript i . By equation (3) the contracted wage for date $\frac{T}{n}$ under the date 0 contract

$$\bar{w}_{\frac{T}{n}}^0 = \bar{w}_0^0 + m\frac{T}{n} \quad (18)$$

$$= w_0 + \frac{1}{2}(\mu + m)\frac{T}{n} \quad (19)$$

is increasing in m . The contracted wage for date $\frac{T}{n}$ under the date $\frac{T}{n}$ will likewise be

$$\bar{w}_{\frac{T}{n}}^{\frac{T}{n}} = w_{\frac{T}{n}}^{\frac{T}{n}} + \frac{1}{2}(\mu - m)\frac{T}{n}$$

We have defined $F(m)$ to be

$$F(m) = \text{Prob}\{\bar{w}_{\frac{T}{n}}^0 \leq \bar{w}_{\frac{T}{n}}^{\frac{T}{n}}\} \quad (20)$$

$$= \text{Prob}\{w_0 + \frac{1}{2}(\mu + m)\frac{T}{n} \leq w_{\frac{T}{n}}^{\frac{T}{n}} + \frac{1}{2}(\mu - m)\frac{T}{n}\} \quad (21)$$

$$= \text{Prob}\{w_{\frac{T}{n}}^{\frac{T}{n}} \geq w_0 + m\frac{T}{n}\} \quad (22)$$

$$= 1 - \Phi\left(\frac{(m-\mu)\sqrt{\frac{T}{n}}}{\sigma}\right) \quad (23)$$

where Φ is the standard normal distribution. Thus $F(m)$ is decreasing in m .

Finally we add in a constraint that as workers oppose wage decreases, the path of wages under the contract is always non-decreasing. We label the shadow price of this constraint λ .

Substituting equation (3), using the properties of a Brownian motion⁷ and integrating, we get as our maximization problem:

$$\min_{n,m} \left\{ \frac{1}{24}n\Delta(\mu - m)^2 \left(\frac{T}{n}\right)^3 + \frac{1}{4}n\Delta\sigma^2 \left(\frac{T}{n}\right)^2 + n[F(m)C + (1 - F(m))\gamma C] - \lambda m \right\}$$

⁶For the sake of simplicity we assume that the wage contract will always lower wages if the optimal initial wage for the new contract is lower than the final wage on the old contract. A more fully specified model would allow for the possibility of keeping the wage the same across contracts but lowering the rate of indexation. As such our equation will tend to slightly overstate the loss.

⁷ $E_0 dZ = 0$ and $E_0 dZ^2 = \sigma^2 t$ as were used in the first minimization problem.

The first-order conditions for this problem are for n and m respectively (given an interior minimum as the second-order conditions are satisfied)

$$-\frac{1}{12}\Delta(\mu - m)^2 \left(\frac{T}{n}\right)^3 - \frac{1}{4}\Delta\sigma^2 \left(\frac{T}{n}\right)^2 + [F(m)C + (1 - F(m))\gamma C] = (04)$$

$$-\frac{1}{12}\Delta(\mu - m) \left(\frac{T}{n}\right)^3 + F'(m)C(1 - \gamma) - \lambda = (05)$$

$$\lambda m = (06)$$

$$m \geq (07)$$

Thus we have two equations in two variables m and $\frac{T}{n}$. Equation (04) is the counterpart to equation (13) from the first minimization problem. In this section we are primarily interested in equation (05), which determines the optimal gap between μ and m . Note that if $\gamma = 1$ then our equations reduce to the first minimization problem with $\mu = m$ and equation (13).

With $\gamma > 1$ we have an asymmetry in the cost structure of negotiations, and this forces $m < \mu$. When μ includes an expected rate of inflation component, this means that we would expect wages to be only partially indexed to inflation during the duration of the contract.

From our first order conditions we have

$$\mu - m = \frac{12F'(m)C(1 - \gamma)}{\Delta \frac{T^3}{n}} - \lambda \geq 0 \quad (28)$$

where this difference represents the degree of partial indexation. For a given contract length, the numerator represents the expected savings in contracting costs allowing w to differ from μ , while the denominator is the expected costs over the length of the contract of having the contract wage differ from the equilibrium wage.

A higher expected deviation from equilibrium wages over the term of the contract is thus traded off against a lower probability of lowering the wage at the end of the contract. If we assume that contracts cannot specify a declining wage over the term of the contract (require $m \geq 0$) then for a rate of inflation close enough to zero m will be constrained to be equal to zero ($\lambda > 0$), and $\mu - m$ will be less than its optimal value. An increase in inflation when inflation is at a low enough level could then actually increase social welfare.

A more complete model of inflation would endogenize the choice of μ by adding a function for the “menu costs” of inflation into equation (05). Assuming that these menu costs are sufficiently negligible

around an inflation rate of zero, the basic result that the optimal inflation rate is greater than zero would be unchanged by the addition of this extra term.

At low levels of inflation, the wage contracts specified under this model accord with the form of contracts we generally see. The wage over the length of the contract will be constant ($m = 0$), and we expect to see large jumps in wages during negotiations.

4 Conclusion

In a wage market with downward rigidity of wages, we show that the optimal wage contracts will specify only partial indexation of wages. We also prove a conjecture of Tobin's that in wage-setting in a dynamic framework, the presence of downward rigidities means that the optimal level of inflation is be non-zero.

References

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